$\frac{10}{10}$

Math 9 Enriched

4.1 Multiplying & Dividing Poly

Name Claire Li | Date 12.17.

1. The polynomial $(3x^2y^3 - 7x^5y^2 + 8y^6)$ is what degree?

   $5+2$

   7

2. The polynomial $(2a - 3ab - 5b + 6)$ is what degree?

   $1+1$

   $2.$

Simplify.

3. $5m^3(3m^2 - 4m)$

   $= 15m^5 - 20m^4$

4. $(5r^2 - 3rt + t^2 - 4r^2t)(6r^2t^2)$

   $= 30r^4t^2 - 18r^3t^3 + 6r^2t^4 - 24r^4t^3.$

5. $-\frac{c}{4}\left(-2c^2 + 8c + \frac{10}{3}\right)$

   $= \frac{c^3}{2} - 2c^2 - \frac{5c}{6}$

6. $\frac{-4a^2 + 12ab - 4ab^2}{-4a}$

   $= a - 3b + b^2$

7. $\frac{27x^3z - 9x^2z^2 + 45x^2z^3}{9x^2z}$

   $= 3x - z + 5z^2$

8. $(14b^3y + 56b^2y^3 - 7b^2y^4) \div (21b^3y^3)$

   $= \frac{2}{3y^2} + \frac{8}{3b} - \frac{y}{3b}$

9. $(6a+1)(4a+1)$

   $= 24a^2 + 6a + 4a + 1$

   $= 24a^2 + 10a + 1$

10. $(12a + c^2)(a + 3c^2)$

    $= 12a^2 + 36ac^2 + ac^2 + 3c^4$

    $= 12a^2 + 37ac^2 + 3c^4$

11. $(c+2)\left(c - \frac{1}{2}\right)$

    $= c^2 - \frac{c}{2} + 2c - 1$

    $= c^2 + \frac{3}{2}c - 1$

12. $(x^2 + x^y)(2x^2 - x^y)$

    $= 2x^4 - x^{y+2} + 2x^{2+y} - x^{2y}$

    $= 2x^4 + x^{y+2} - x^{2y}$

$$\begin{cases}(a+b)^3 = a^3+3a^2b+3ab^2+b^3 \\ (a+b+c)^2 = a^2+b^2+c^2+2ab+2ac+2bc\end{cases}$$

13. $(a^b + a^c)(a^b + a^c)$

$= (a^b+a^c)^2$

$= a^{2b}+2a^{b+c}+a^{2c}$

14. $(a^2 - 5)(a^2 + 5)$

$= (a^2)^2 - 5^2$

$= a^4 - 25$

15. $(b - c^3)(b + c^3)$

$= b^2 - c^6$

16. $(x^a - 4x)(x^a + 4x)$

$= x^{2a} - 16x^2$

17. $(a^{3x} + b^{5x})(a^{3x} - b^{5x})$

$= a^{6x} - b^{10x}$

18. $(a^{n+1} - b^n)(a^{n+1} + b^n)$

$= a^{2n+2} - b^{2n}$

19. $(x - y)(3x^2 + 2xy - 5y + 2y)$

$= 3x^3+2x^2y-3xy-3x^2y+2xy^2+3y^2$

$= 3x^3-x^2y+2xy^2-3xy+3y^2$

20. $(4x + y)(2x + y - 3xy - 8y^2)$

$= 8x^2+4xy-12x^2y-32xy^2+2xy+y^2-3xy^2-8y^3$

$= 8x^2+6xy-35xy^2-12x^2y+y^2-8y^3$

21. $(2y^2 + 5y - 3)^2$

$= (2y^2)^2+(5y)^2+3^2+20y^3-12y^2-15y$

$= 4y^4+25y^2+9+20y^3-12y^2-15y$

$= 4y^4+20y^3+13y^2-15y+9$

22. $(2x^2 - 3xy + y^2)(3x^2 + 5xy - 4y^2)$

$= 6x^4+\underline{10x^3y}-8x^2y^2-\underline{9x^3y}-15x^2y^2+\underline{12xy^3}+3x^2y^2$

$= 6x^4+x^3y-26x^2y^2+17xy^3-4y^4$ $\quad +5xy^3-4y^4$

23. $(c - 4)^3$

$= c^3-12c^2+48c-64$

24. $(a + 1)(a^2 - a + 1)$

$= a^3-a^2+a+a^2-a+1$

$= a^3-2a^2+1$

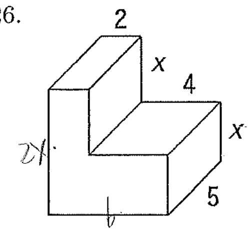
25. $(4 + k)(16 + 4k + k^2)$

$= 64+16k+\underline{4k^2}+16k+\underline{4k^2}+k^3$

$= k^3+8k^2+32k+64$

Write a polynomial for the volume of the figure.

26.

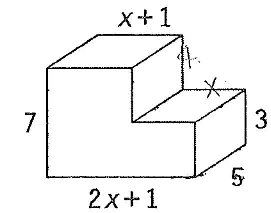


$2x\cdot 6\cdot 5 - x\cdot 4\cdot 5$

$= 60x - 20x$

$= \boxed{40x}$ //

27.



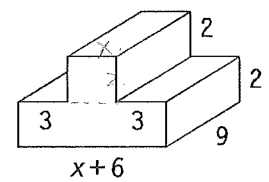
$5[7(2x+1)-4x]$

$=5(14x+7-4x)$

$=5(10x+7)$

$=\boxed{50x+35}$ //

28.



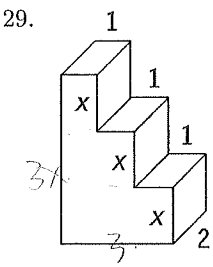
$9[2(x+6)+2x]$

$=9(2x+12+2x)$

$=9(4x+12)$

$=\boxed{36x+108}$ //

29.

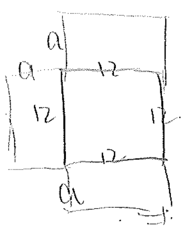


$2(3x\cdot 3-3\cdot x)$

$=2\cdot 6x$

$=\boxed{12x}$ //

30. The side of a square is 12. A strip of uniform width is added to three sides. What is the new area and perimeter? let the width be a.



Area $= a\cdot 12\cdot 3+12^2$

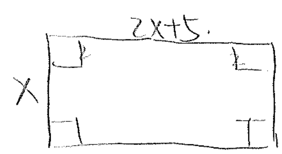
$=\boxed{36a+144}$ //

Perimeter $= a\cdot 6+12\times 4$

$=\boxed{6a+48}$ //

31. The length of a rectangle is 5 more than twice the width. It is made into a box by cutting out four squares from the corners. The side of each square is 2. What is the volume of the box?

Let the width be x.

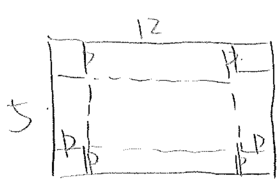


$V=(2x+5-4)(x-4)2$

$=(2x^2-8x+x-4)2$

$=\boxed{4x^2-14x-8}$ //

32. A rectangular piece of cardboard has dimensions 12×5 cm. The cardboard is folded into box by cutting squares from the corners. The side of each square is p. What is the volume of the box?



$(12-2p)(5-2p)\cdot p$

$=(60-24p-10p+4p^2)p$

$=(60p-34p^2+4p^3)\text{cm}^3$

33. A rectangular piece of cardboard has dimensions 10×8 cm. The cardboard is folded into box by cutting squares from the corners. The side of each square is r. What is the volume of the box?

$V=(10-2r)(8-2r)r$

$=(80-20r-16r+4r^2)r$

$=(80r-36r^2+4r^3)\text{cm}^3$

34. If $(x+2)(3x^2-x+5) = Ax^3+Bx^2+Cx+D$, what is the value of $A+B+C+D$?

$(x+2)(3x^2-x+5)$

$= 3x^3-x^2+5x+6x^2-2x+10$

$= 3x^3+5x^2+3x+10$

$A=3,\ B=5,\ C=3,\ D=10$

$A+B+C+D = \boxed{21}$ //

35. Find the smallest real value of x that satisfies the equation:

$$(x+5)(x^2-x-11) = x+5$$

1) $x+5=0$, $x=-5$

2) $x^2-x-11=1$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$x=4$ or -3

$\therefore x_{min}=-5$

36. The formula for finding the sum of the first n positive integers is given by $S = \dfrac{n(n+1)}{2}$, where S is the sum. What is the sum of $42+43+44+\cdots+78+79$?

$\dfrac{(42+79)(79-42+1)}{2}$

$= \dfrac{121 \times 38}{2} = 121 \times 19 = \boxed{2299}$ //

37. The formula for finding the sum of the squares of the first n positive integers is given by $S = \dfrac{n}{6}(n+1)(2n+1)$, where S is the sum. What is the sum of $1^2+2^2+3^2+\cdots+18^2+19^2$?

$\dfrac{19}{6}(19+1)(2\cdot 19+1)$

$= \dfrac{19 \cdot \cancel{20}^{10} \cdot \cancel{39}^{13}}{\cancel{6}} = \boxed{2470}$ //

38. The formula for finding the sum of the squares of the first n positive integers is given by $S = \dfrac{n}{6}(n+1)(2n+1)$, where S is the sum. What is the sum of $16^2+17^2+18^2+\cdots+31^2+32^2$?

$\dfrac{32}{6}(32+1)(64+1) - \dfrac{15}{6}(15+1)(30+1)$

$= \dfrac{32 \cdot 33 \cdot 65}{6} - \dfrac{15 \cdot 16 \cdot 31}{6}$

$= 11440 - 1240 = \boxed{10200}$ //

40. The formula for finding the sum of the cubes of the first n positive integers is given by $S = \dfrac{n^2}{4}(n+1)^2$, where S is the sum. What is the sum of $11^3+12^3+13^3+\cdots+23^3+24^3$?

$\dfrac{24^2}{4}(24+1)^2 - \dfrac{10^2}{4}(10+1)^2$

$= \dfrac{24^2 \cdot 25^2}{4} - \dfrac{10^2 \cdot 11^2}{4}$

$= \dfrac{600^2 - 110^2}{4}$

$= \dfrac{(600-110)(600+110)}{4}$

$= \dfrac{490 \cdot 710}{4} = \boxed{86975}$ //

39. The formula for finding the sum of the cubes of the first n positive integers is given by $S = \dfrac{n^2}{4}(n+1)^2$, where S is the sum. What is the sum of $1^3+2^3+3^3+\cdots+13^3+14^3$?

$S = \dfrac{14^2}{4}(14+1)^2$

$= \dfrac{14^2 \cdot 15^2}{14}$

$= \dfrac{210^2}{14}$

$= 2250$

Math 9 Enriched   4.1 Multiplying & Dividing Poly   Mr. Young   10/22/2011

Answer List

1. 6
2. 1
3. $15m^5 - 20m^4$
4. $30r^4t^2 - 18r^3t^3 + 6r^2t^4 - 24r^4t^3$
5. $\frac{c^3}{2} - 2c^2 - \frac{5c}{6}$
6. $a - 3b + b^2$
7. $3x - z + 5z^2$
8. $\frac{2}{3y^2} + \frac{8}{3b} - \frac{y}{3b}$
9. $24a^2 + 10a + 1$
10. $12a^2 + 37ac^2 + 3c^4$
11. $c^2 + \frac{3}{2}c - 1$
12. $2x^4 + x^{y+2} - x^{2y}$
13. $a^{2b} + 2a^{b+c} + a^{2c}$
14. $a^4 - 25$
15. $b^2 - c^6$
16. $x^{2a} - 16x^2$
17. $a^{6x} - b^{10x}$
18. $a^{2n+2} - b^{2n}$
19. $3x^3 - x^2y - 2xy^2 + 3y^2 - 3xy$
20. $-8y^3 - 12x^2y - 35xy^2 + 8x^2 + y^2 + 6xy$
21. $4y^4 + 20y^3 + 13y^2 - 30y + 9$
22. $6x^4 + x^3y - 20x^2y^2 + 17xy^3 - 4y^4$
23. $c^3 - 12c^2 + 48c - 64$
24. $a^3 - 1$
25. $64 + k^3$
26. $A = 36x + 60,\ V = 40x$
27. $A = 24x + 160,\ V = 32x + 112$
28. $A = 26x + 204,\ V = 36x + 108$
29. $A = 22x + 216,\ V = 24x + 160$
30. $144 + 36w + 2w^2;\ 48 + 6w$
31. $4w^2 - 14w - 8$
32. $60p - 34p^2 + 4p^3$
33. $80r - 36r^2 + 4r^3$
34. 21
35. -5
36. 2299
37. 2470
38. 10200
39. 11025
40. 86975

Catalog List

1. ALG EL 7
2. ALG EL 8
3. ALG EC 13
4. ALG EC 93
5. ALG EC 115
6. ALG EH 35
7. ALG EH 43
8. ALG EH 168
9. ALG ED 122
10. ALG ED 161
11. ALG ED 175
12. ALG ED 236
13. ALG ED 235
14. ALG EF 42
15. ALG EF 50
16. ALG EF 82
17. ALG EF 86
18. ALG EF 88
19. ALG EG 58
20. ALG EG 60
21. ALG EG 78
22. ALG EG 86
23. ALG EG 111
24. ALG EG 133
25. ALG EG 140
26. ALG EM 9
27. ALG EM 11
28. ALG EM 15
29. ALG EM 18
30. ALG EK 54
31. ALG EK 57
32. ALG EK 59
33. ALG EK 60
34. MCC BA 40
35. MCC BC 164
36. CM2 ED 65
37. CM2 ED 67
38. CM2 ED 69
39. CM2 ED 71
40. CM2 ED 73

ame: Claire Date: 1.6.

Math 9 Honors: Section 4.2 Binomial Expansions:

1. Expand the following expressions using Pascal's Triangle

a) $(x+a)^3 = x^3+3x^2a+3xa^2+a^3$

b) $(x-a)^4 = x^4-4x^3a+6x^2a^2-4xa^3+a^4$

c) $(x+a)^5 = x^5+5x^4a+10x^3a^2+10x^2a^3+5xa^4+a^5$

d) $(\sqrt{3}+a)^6 = \sqrt{3}^6+6\cdot\sqrt{3}^5a+15\cdot\sqrt{3}^4a^2+20\cdot\sqrt{3}^3a^3+15\cdot\sqrt{3}^2a^4+6\cdot\sqrt{3}\cdot a^5+a^6$

$= 27+54\sqrt{3}a+135a^2+60\sqrt{3}a^3+45a^4+6\sqrt{3}a^5+a^6$

e) $\left(2-\frac{1}{a}\right)^3 = 2^3-3\cdot2^2\cdot\frac{1}{a}+3\cdot2\left(\frac{1}{a}\right)^2-\left(\frac{1}{a}\right)^3$

$= 8-\frac{12}{a}+\frac{6}{a^2}-\frac{1}{a^3}$

f) $(\sqrt{2}-1)^5$ (Expand and find the exact value without a calculator)

$= \sqrt{2}^5-5\sqrt{2}^4\cdot1+10\sqrt{2}^3-10\sqrt{2}^2+5\sqrt{2}-1$

$= 4\sqrt{2}-20+20\sqrt{2}-20+5\sqrt{2}-1 = \boxed{29\sqrt{2}-41}$

2. Expand the following expression and indicate what the constant term is:

i) $\left(x^2+\frac{1}{x}\right)^5 = x^{10}+5x^8\cdot\frac{1}{x}+10x^6\cdot\frac{1}{x^2}+10x^4\cdot\frac{1}{x^3}+5x^2\cdot\frac{1}{x^4}+\frac{1}{x^5}$

$= x^{10}+5x^7+10x^4+10x+5\frac{1}{x^2}+\frac{1}{x^5}$

No constant term.

ii) $\left(2x^3-\frac{1}{x^2}\right)^{10} = (2x^3)^{10}+10(2x^3)^9\left(-\frac{1}{x^2}\right)+45(2x^3)^8\left(-\frac{1}{x^2}\right)^2+120(2x^3)^7\left(-\frac{1}{x^2}\right)^3+210(2x^3)^6\left(-\frac{1}{x^2}\right)^4+252(2x^3)^5\left(-\frac{1}{x^2}\right)^5+$

$210(2x^3)^4\left(-\frac{1}{x^2}\right)^6+120(2x^3)^3\left(-\frac{1}{x^2}\right)^7+45(2x^3)^2\left(-\frac{1}{x^2}\right)^8+10(2x^3)\left(-\frac{1}{x^2}\right)^9+\left(-\frac{1}{x^2}\right)^{10}$

$= 1024x^{30}-5120x^{25}+11520x^{20}-15360x^{15}+13440x^{10}-8064x^5+\boxed{3360}-960\frac{1}{x^5}+180\frac{1}{x^{10}}-10\frac{1}{x^{15}}+\frac{1}{x^{20}}$

Constant term: 3360.

iii) $\left(x^4-\frac{2}{x}\right)^4$

$= x^{16}-4x^{12}\cdot\frac{2}{x}+6x^8\cdot\frac{4}{x^2}-4x^4\cdot\frac{8}{x^3}+\frac{16}{x^4}$

$= x^{16}-8x^{11}+24x^6-32x+\frac{16}{x^4}$

No constant term.

3. Suppose that the number "x" satisfies the equation: $5=x+x^{-1}$. What is the value of x^4+x^{-4}?

$(x+\frac{1}{x})^4=x^4+4x^3\cdot\frac{1}{x}+6x^2\cdot\frac{1}{x^2}+4x\cdot\frac{1}{x^3}+\frac{1}{x^4}$ $\qquad (x+\frac{1}{x})^2=x^2+2$

$5^4=x^4+x^{-4}+4x^2+4x^{-2}+6$ $\qquad x^4+x^{-4}=\boxed{527}$ $\qquad 25=x^2+x^{-2}+2$

$625=x^4+x^{-4}+92+6$ $\qquad 23=x^2+x^{-2}$

4. There are integers "a" and "b" such that: $(1+\sqrt{2})^{16}=a+b\sqrt{2}$. What is the value of "a"?

$=1+16\sqrt2+{}_{16}C_{14}\cdot2+{}_{16}C_{13}\cdot2\sqrt2+{}_{16}C_{12}\cdot4+{}_{16}C_{11}\cdot4\sqrt2+{}_{16}C_{10}\cdot8+{}_{16}C_{9}\cdot8\sqrt2+{}_{16}C_{8}\cdot16+{}_{16}C_{7}\cdot16\sqrt2+{}_{16}C_{6}\cdot32+$
${}_{16}C_{5}\cdot32\sqrt2+{}_{16}C_{4}\cdot64+{}_{16}C_{3}\cdot64\sqrt2+{}_{16}C_{2}\cdot128$

$a=\boxed{665857}$ $=1+16\sqrt2+240+1120\sqrt2+7280+17472\sqrt2+64064+91520\sqrt2+205920+16\cdot128\sqrt2+256$
$183040\sqrt2+256256+139776\sqrt2+116480+35840\sqrt2+15360\sqrt2\ 256$

5. Given the expression: $(x+a)^{\ }$, what are the values of "a" and "x" if the third term of the expression is equal to 4320 and the fifth term is equal to 3840.

$10x^3a^2=4320$ $\qquad x^3a^2=432$ $\qquad 432=2^4\cdot3^3$

$5xa^4=3840$ $\qquad xa^4=768$ $\qquad 768=2^8\cdot3$

$\boxed{\begin{cases}x=3\\a=4\end{cases}}$

6. Given the expression: $(a-b)^4$, what are the values of "a" and "b" if the third term of the expression is equal to 108 and the fourth term is $-24\sqrt{2}$.

$-{}_4C_3ab^3=-24\sqrt2$

$\begin{cases}{}_4C_2a^2b^2=108\\{}_4C_3ab^3=-24\sqrt2\end{cases}$ $\rightarrow {}_4C_2a^2b^2=108$ $\qquad ab^3=6\sqrt2$

$6a^2b^2=108$ $\qquad ab^3=3\cdot(\sqrt2)^3$

$a^2b^2=18$ $\qquad \therefore \boxed{\begin{cases}a=3\\b=\sqrt2\end{cases}}$

$a^2b^2=3^2\cdot(\sqrt2)^2$

7. If the first term of the expansion $(a+b)^6$ is equal to 512 and the last term is 5832, then what is the value of the expression?

$=a^6+6a^5b+15a^4b^2+20a^3b^3+15a^2b^4+6ab^5+b^6$

$a^6=512$ $\qquad b^6=5832$ $\qquad =512+6\cdot2^5\cdot4\cdot\sqrt2\cdot3\sqrt2+15\cdot64\cdot18+20\cdot(6\cdot2)^3+15\cdot8\cdot18^2+6\cdot2\sqrt2\cdot3^5$

$a=2\sqrt[6]{8}$ $\qquad b^6=2^3\cdot3^6$ $\qquad =512+4608+17280+1748+38880+23328+5832$ $\qquad 4\sqrt2$ $+5832$

$a=2\sqrt2$ $\qquad b=3\sqrt2$ $\qquad =\boxed{92188}$

8. If the sum of the coefficients of $(a+b)^n$ is equal to 128, how many terms are in the expansion?

$128=2^7$

$7+1=\boxed{8}$ terms.

9. If $(x+3)^5-(x+2)^4=Ax^5+Bx^4+Cx^3+Dx^2+Ex+F$. What is the value of $A+B+C+D+E+F$?

$=x^5+5x^4\cdot3+10x^3\cdot3^2+10x^2\cdot3^3+5x\cdot3^4+3^5-(x^4+4x^3\cdot2+6x^2\cdot4+4x\cdot8+16)$

$=x^5+15x^4+90x^3+270x^2+405x+243-x^4-8x^3-24x^2-32x-16$

$=x^5+14x^4+82x^3+246x^2+373x+227$

$1+14+82+246+373+227=\boxed{1003}$

10. The 4[th] term of $(x-0.5)^n$ is $-15x^7$. What is the value of "n"?

${}_nC_3\cdot x^{n-3}(-0.5)^3=-15x^7$ $\qquad \frac{1}{8}\cdot{}_{10}C_3=\frac{1}{8}\cdot\frac{10\cdot9\cdot8}{3\cdot2}=15$ ✓

$-\frac{1}{8}\cdot{}_nC_3\cdot x^{n-3}=-15x^7$ $\qquad n=\boxed{10}$

$\begin{cases}\frac{1}{8}\cdot{}_nC_3=15\\x^{n-3}=x^7\end{cases}\Rightarrow n-3=7$

$n=10$

11. The 7th term of $(2x-1)^n$ is $112x^2$. What is the value of "n"?

$n = 7+2-1 = \boxed{8}$ //

12. Determine "b" such that $(x-b)^{10}$ has the term $-1875x^7$

$10-7+1 = 4$th term

$_{10}C_3 \cdot x^7 \cdot (-b)^3 = -1875$

$120 \cdot b^3 = 1875$

$b^3 = \frac{125}{8}$

$b = \boxed{\frac{5}{2}}$ //

13. $(\sqrt{3}-1)^4 - (2\sqrt{3}+2)^3 = A + B\sqrt{C}$. What is the value of $A+B+C$?

$= \sqrt{3}^4 - 4\sqrt{3}^3 + 6\sqrt{3}^2 - 4\sqrt{3} + 1 - \left((2\sqrt{3})^3 - 3(2\sqrt{3})^2 \cdot 2 - 3 \cdot 2\sqrt{3} \cdot 2^2 - 2^3\right)$

$= 9 - 12\sqrt{3} + 18 - 4\sqrt{3} + 1 - 6\sqrt{3} - 72 - 24\sqrt{3} - 8$

$= -52 - 46\sqrt{3}$

$-52 + (-46) + 3 = \boxed{-95}$ //

14. Solve for all the possible value(s) of "x" $\quad \dfrac{x^3+3x^2+3x+1}{x+1} = 25$

$\frac{(x+1)^3}{x+1} = 25$

$(x+1)^2 = 25$

$x+1 = \pm 5$

$x = \boxed{4 \text{ or } -6}$ //

15. Solve for all the possible value(s) of "x" $\quad \dfrac{x^5-5x^4+10x^3-10x^2+5x-1}{(1-x)^2} = 343$

$\frac{(x-1)^5}{(x-1)^2} = 343$

$(x-1)^3 = 7^3$

$x-1 = 7$

$x = \boxed{8}$ //

16. The expansion of $(3+2x)(1-x)^n = ax^2 - 10x + 3$ Find the values of "a" and "n"

$(3+2x)(1-x)$

$= 3 - 3x + 2x - 2x^2$

$= -2x^2 - x + 3$

$-2x^2 - x = ax^2 - 10x$

$x(2x+1) = x(10 - ax)$

$2x+1 = 10 - ax$

$(a+2)x = 9$

$\begin{cases} a+2=3 \\ x=3 \end{cases}$ or $\begin{cases} a+2=9 \\ x=1 \end{cases}$ or

$\therefore \begin{cases} a=1 \\ x=3 \end{cases}$ or $\begin{cases} a=7 \\ x=1 \end{cases}$ or $\begin{cases} a=-1 \\ x=9 \end{cases}$ $\begin{cases} a+2=1 \\ x=9 \end{cases}$

17. Challenge: Find the coefficient of x^2 in the expression: $(2+x-x^2)^4 - (3-2x-x^2)^5$

$= (x^2-x-2)^4 + (x^2+2x-3)^5$

$= (x-2)^4(x+1)^4 + (x+3)^5(x-1)^5$

$(x-2)^4(x+1)^4$

$(x^4 - 8x^3 + 24x^2 - 32x + 16)(x^4 + 4x^3 + 6x^2 + 4x + 1)$

$24x^2 - 128x^2 + 96x^2 = -8x^2$

$(\cdots + 10x^2 \cdot 3^3 + 5x \cdot 3^4 + 3^5)(\cdots - 10x^2 + 5x - 1)$

$-90x^2 + 2025x^2 - 2430x^2 = -495x^2$

$= -8x^2 - 495x^2$

$= \boxed{-503}x^2$ //

Name: Claire   Date: 1.8

Section 4.3 Factoring Difference of Squares and Cubes

Difference of squares: $a^2-b^2=(a+b)(a-b)$

Difference and Sums of Cubes: $x^3+y^3=(x+y)(x^2-xy+y^2)$ $x^3-y^3=(x-y)(x^2+xy+y^2)$

Difference of Powers: $x^n-y^n=(x-y)(x^{n-1}+x^{n-2}y+\ldots+xy^{n-2}+y^{n-1})$

Difference of Sums: $x^{2n+1}+y^{2n+1}=(x+y)(x^{2n}-x^{2n-1}y+x^{2n-2}y^2\ldots-xy^{2n-1}+y^{2n})$

1. Factor each of the following expressions:

a) $16x^2-49y^2$ $=(4x-7y)(4x+7y)$	b) $(2x-1)^2-9x^2$ $=(2x-1-3x)(2x-1+3x)$ $=-(x+1)(5x-1)$ $=(x+1)(1-5x)$	c) $81a^2-(3a+2b)^2$ $=(9a-3a-2b)(9a+3a+2b)$ $=(6a-2b)(12a+2b)$ $=4(3a-b)(6a-b)$
d) $4(2x-y)^2-25z^2$ $=(4x-2y-5z)(4x-2y+5z)$	e) $18x^2y^2-50y^4$ $=2(3xy-5y^2)(3xy+5y^2)$	f) $\frac{x^2}{16}-\frac{y^2}{49}=(\frac{x}{4}-\frac{y}{7})(\frac{x}{4}+\frac{y}{7})$
g) $5x^4-80$ $=5(x^4-16)$ $=5(x^2-4)(x^2+4)$ $=5(x-2)(x+2)(x^2+4)$	h) $(3x-4)^3+(x+3)^3$ $+(x+3)^2]$ $=(3x-4+x+3)[(3x-4)^2+(3x-4)(x+3)$ $=(4x-1)(9x^2-24x+16+3x^2+5x-12+x^2+6x+9)$ $=(4x-1)(13x^2-13x+13)$ $=13(4x-1)(x^2-x+1)$	i) $(4x^2-4)^2-81x^4$ $=(4x^2-4-9x^2)(4x^2-4+9x^2)$ $=-(5x^2+4)(13x^2-4)$ $=(5x^2+4)(4-13x^2)$
j) $a^4-16a^6b^2$ $=(a^2-4a^3b)(a^2+4a^3b)$	k) a^6+7a^3-8 $=(a^3+8)(a^3-1)$ $=(a+2)(a^2-2a+4)$	L) $3a^4-15a^2-108$ $=3(a^4-5a^2-36)$ $=3(a^2-9)(a^2+4)$ $=3(a-3)(a+3)(a^2+4)$
m) $27x^3+64y^6$ $=(3x)^3+(4y^2)^3$ $=(3x+4y^2)(9x^2-12xy^2+16y^4)$	n) $125x^3-8x^6$ $=(5x-2x^2)(25x^2-10x^3+4x^4)$ $=x^3(5-2x)(25-10x+4x^2)$	o) $125x^6+64x^9$ $=x^6(125+64x^3)$ $=x^6(5+4x)(25+20x+16x^2)$

2. For what value of "n" does $(2^{2007}-2^{2006})(2^{1997}-2^{1996})=2^n$?

$2^{2006}(2-1)\cdot 2^{1996}(2-1)=2^n$ $n=4002$

$2^{2006+1996}=2^n$

3. The number 2001 can be written as a difference of squares, x^2-y^2 where "x" and "y" are positive integers in four different ways. What are the four possible ways? $(x-y)(x+y)$

$2001=3\cdot 23\cdot 29$

$\begin{cases}x-y=3\\x+y=667\end{cases}$ $\begin{cases}x-y=23\\x+y=87\end{cases}$ $\begin{cases}x-y=29\\x+y=69\end{cases}$ $\begin{cases}x-y=1\\x+y=2001\end{cases}$

$x=335, y=332$ $x=55, y=32$ $x=49, y=20$ $x=1001, y=1000$

4. Two numbers are such that their difference, their sum and their product are to one another as 1 : 7 : 18. The product of the two numbers are: $(x-y)$ $x+y$ xy $1k:7k:18k$

a) 6 b) 12 c) 24 d) 48 e) none of these

$\begin{cases}x-y=1k\\x+y=7k\end{cases}$

$x=4k, y=3k$

$xy=12k^2$

$18k=12k^2$

$\frac{3}{2}=k$

$9\cdot 18\cdot\frac{3}{2}=\boxed{27}$

5. The number 2005 can be written in the form of $a^2 - b^2$, where "a" and "b" are positive integers less than 1000 in exactly one way. What is the value of $a^2 + b^2$?

$203^2 + 198^2 = 80413.$

$2005 = 5 \cdot 401.$ $\begin{cases} a-b=5 \\ a+b=401. \end{cases}$

$a = 203, \quad b = 198.$

6. Solve for "a" and "b" in the expression: $1+\sqrt{8} = \left(a+\sqrt{2}\right)\left(b-\sqrt{2}\right)$

$= 1^3 + \sqrt{2}^3$

$= (1+\sqrt{2})(1-\sqrt{2}+2)$

$= (1+\sqrt{2})(3-\sqrt{2})$

$\therefore \begin{cases} a=1. \\ b=3. \end{cases}$

7. Given that "p" is a prime number, solve for "x": $1000 - 8x^3 = 8(p)(5p+2)$

$8(125-x^3) = 8(p)(5p+2).$

$(5-x)(25+5x+x^2) = p(5p+2)$

$\therefore \begin{cases} 5-x=p. \\ 25-5x+x^2=5p+2 \end{cases}$ or $\begin{cases} 5-x=5p+2. \\ 25-5x+x^2=p. \end{cases}$

$25-5x+x^2 = 5(5-x)+2.$

$x^2-5x+25 = 25-5x+2.$

$x^2 - 2.$

$x = 0$ $\quad x = \pm\sqrt{2}$

$p = \frac{3-x}{5}.$

$25-5x+x^2 = \frac{3-x}{5}$

$5x^2-26x-122=0$

8. Evaluate : $\dfrac{2^{10}-1}{2^5-1}$

$= \dfrac{(2-1)(2^9+2^8+2^7+2^6+2^5+2^4+2^3+2^2+1)}{(2-1)(2^4+2^3+2^2+2^1)}$

$= 2^5+1+\dfrac{32}{16+8+4+2} = \boxed{34\frac{1}{15}}$

9. Suppose that $n^2 - 4 = 50(n-2)$ and "n" is not equal to 2. What is the value of "n"?

$(n-2)(n+2) = 50(n-2)$

$n+2=50$

$n = \boxed{48}$

10. Find as many prime numbers "p" as you can so that the expression 5p+1 is a perfect square (A^2). How many prime numbers like this do you think there are? Prove that there are only this many primes.

$5p+1 = A^2$

$5p = A^2-1$

$5p = (A-1)(A+1)$

$p=7.$

11. The positive difference of two perfect squares is 32. What is the largest possible value of the sum of the two perfect squares?

$a^2-b^2=32.$

$(a+b)(a-b) = 32 \times 1$

$\therefore \begin{cases} a=\frac{33}{2} \\ b=\frac{31}{2} \end{cases}$

$\left(\frac{33}{2}\right)^2 + \left(\frac{31}{2}\right)^2 = 272.25 + 240.25$

$= \boxed{512.5}$

12. How many ordered pairs (m,n) of positive integers, with m>n, have the property that their squares differ by 96? List out all possible pairs.

$m^2 - n^2 = 96$

$(m+n)(m-n) = 96 \times 1$

$= 48 \times 2$

$= 32 \times 3$

$= 24 \times 4$

$= 16 \times 6$

$= 12 \times 8$

$\therefore \begin{cases} m=\frac{97}{2} \\ n=\frac{95}{2} \end{cases}$ $\begin{cases} m=25 \\ n=23 \end{cases}$ $\begin{cases} m=\frac{35}{2} \\ n=\frac{29}{2} \end{cases}$ $\begin{cases} m=14 \\ n=10 \end{cases}$ $\begin{cases} m=11 \\ n=5 \end{cases}$ $\begin{cases} m=10 \\ n=2 \end{cases}$

13. Given that "p" is a prime number and the expression 2003p + 16 is a perfect square, what is the lowest possible value of "p"?

$(a+4)^2 = a^2 + 8a + 16$

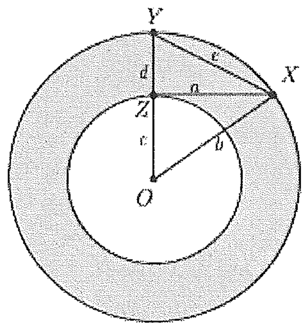
$2003p + 16 = a^2 + 8a + 16$

$2003p = a(a+8)$

$\begin{cases} p = 2011 \\ a = 2003 \end{cases}$

14. An annulus is the region between two concentric circles. The concentric circles in the figure have radii "b" and "c", with b > c. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let $a = XZ$, $d = YZ$, and $e = XY$. What is the area of the annulus?

(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2



$b^2\pi - c^2\pi$

$= (b^2 - c^2)\pi$

$= a^2\pi$

15. If $a^2 - b^2 = 64$, what is the smallest possible values of $(a+b)$?

$(a-b)(a+b) = 64$

$(a-b)(a+b) = 8 \times 8$

$a+b = \boxed{8}$ //

16. There are four different positive integers "a", "b", "c" , and "d" such that the equation is true: $a^3 + b^3 = c^3 + d^3 = 1729$. What is the values of $a + b + c + d$?

$(a+b)(a^2 - ab + b) = (c+d)(c^2 - cd + d^2) = 7 \cdot 13 \cdot 19$

$\begin{cases} a = 1 \\ b = 12 \end{cases}$ and $\begin{cases} c = 9 \\ d = 10 \end{cases}$

$1 + 12 + 9 + 10 = \boxed{32}$ //

17. What is the sum of the (decimal) digits of $10^6 - 8$?

$100^3 - 2^3$

$= (100-2)(100^2 - 200 + 64)$

$= 98 \cdot 9864$

$= 966672$

$9+6+6+6+7+2 = \boxed{36}$ //

18. What are both pairs of integers (x,y) for which $4^y - 615 = x^2$

$(2^y)^2 - x^2 = 615$

$(2^y - x)(2^y + x) = 615$

$(2^y - x)(x^y + x) = 3 \cdot 5 \cdot 41$

19. Show that if the positive integer "n" is a multiple of 3, then $7^n - 6^n$ is a multiple of 127.

$n = 3k$

$7^n - 6^n$

$= (7^k)^3 - (6^k)^3$

$= (7^k - 6^k)(7^{2k} + 42^k + 6^{2k})$

Remainder 1·8

20. Show that $2^{99}+3^{99}$ is divisible by 35.

$2^{99} = 2^{4(24)} \cdot 2^3$

$+\ 3^{99} = 3^{4(24)} \cdot 3^3$

$\phi(35)=24 \quad \gcd(x,35)=1$

$x^{\phi(35)} \equiv 1 \pmod{35}$

Remainder $\boxed{35}$

$2^{99}+3^{99} \qquad 99 = 4(24)+3$

$1 \cdot 27$

21. Let "a" and "b" be the roots of the equation $x^2-mx+2=0$. Suppose that $a+\frac{1}{b}$ and $b+\frac{1}{a}$ are the roots of the equation: $x^2-px+q=0$. What is the value of q?

Vieta's theorem

$ab=2$

$(a+\frac{1}{b})(b+\frac{1}{a})=q \qquad q=\boxed{\frac{9}{2}}$

$ab+1+1+\frac{1}{ab}=q$

$4+\frac{1}{2}=q$

22. Challenge: March 2009 (Adler). Show that $n^{n-1}-1$ is divisible by $(n-1)^2$ for every positive integer "n".

$=(n-1)(n^{n-2}+n^{n-3}+n^{n-4}+\cdots+n+1)$

23. (Challenge) Let "x" and "y" be two-digit integers such that "y" is obtained by reversing the digits of "x". The integers "x" and "y" satisfy $x^2-y^2=m^2$ for some positive integer "m". What is the value of $x+y+m$?

Let x be $\overline{ab}$, y be $\overline{ba}$.

$x^2-y^2=$

$=(x-y)(x+y)$

$=(10a+b-10b-a)(10a+b+10b+a)$

$=(9a-9b)(11a+b)$

$=3^2 \cdot 11(a-b)(a+b)$

$65+56+33=154$

$\begin{cases} a+b=11 \\ a-b=1 \end{cases} \Rightarrow \begin{cases} a=6 \\ b=5 \end{cases}$

$\therefore x=65,\ y=56,\ m=33.$

Name: Claire Date: 1.10

Math 9 Enriched: Section 4.4 Factoring & Solving Trinomials

1. Factor each of the following expressions:

a. $2x^2+5x+2$
$=(2x+1)(x+2)$

b. $4x^2+9x+2$
$=(4x+1)(x+2)$

c. $21x^2+17x-30$
$=21x^2+7x+10x-30$
$=7x(3x+1)+10(x-3)$

d. $2x^2-11x+15$
$=(2x-5)(x-3)$

e. $8x^4-14x^2y^2+3y^4$
$=(2x^2-3y^2)(4x^2-y^2)$
$=(2x^2-3y^2)(2x-y)(2x+y)$

f. $15x^2-28x-32$
$=(3x+8)(5x-4)$

g. $2m^3n-m^2n-21mn$
$=mn(2m^2-m-21)$
$=mn(2m+7)(m-3)$

h. $10x^2-33xy-7y^2$
$=(2x-7y)(5x+y)$

i) $16mn-4m^2+28n-7m$
$=4m(4n-m)+7(4n-m)$
$=(4m+7)(4n-m)$

j) $4xy+6-x-24y$
$=(6-x)-(24y-4xy)$
$=(6-x)-4y(6-x)$
$=(1-4y)(6-x)$

k) $21xy-12b^2+14xb-18by$
$=(14xb+21xy)+(12b^2+18by)$
$=7x(2b+3y)-6b(2b+3y)$
$=(7x-6b)(2b+3y)$

L) $28xy+25+35x+20y$
$(28xy+35x)+(20y+25)$
$7x(4y+5)+5(4y+5)$
$=(7x+5)(4y+5)$

2. Factor and solve each of the following expressions:

a. $x^2-13x+40=0$
$(x-5)(x-8)=0$
$x=5$ or 8

b. $x^2-x+56=-13$
$x^2-x+69=0$
$\Delta=\sqrt{1-4\cdot69}<0$
No solution.

c. $x^3+4x^2-12x=0$
$x(x^2+4x-12)=0$
$x(x+6)(x-2)=0$
$x=0$ or -6 or 2

d. $x(x+2)=2x(x-3)$
$x^2+2x=2x^2-6x$
$0=x^2-8x$
$0=x(x-8)$
$x=0$ or 8

e. $x=x^2(x+5)$
$0=x^3+5x^2-x$
$0=x(x^2+5x-1)$
$x=0$ or $\frac{-5-\sqrt{29}}{2}$ or $\frac{-5+\sqrt{29}}{2}$

f. $(x+2)=(x+2)(x-4)$
$x+2=x^2-2x-8$
$0=x^2-3x-10$
$0=(x-5)(x+2)$
$x=5$ or -2

g) $x^3+4x^2-12x=0$
$x(x^2+4x-12)=0$
$x(x+6)(x-2)=0$
$x=0$ or 2 or -6

h) $2y^3+6y^2-108y=0$
$2y(y^2+3y-54)=0$
$2y(y-9)(y+6)=0$
$y=0$ or 9 or -6

i) $12y^3-21y^2+28y-49=0$
$3y^2(4y-7)+7(4y-7)=0$
$(3y^2+7)(4y-7)=0$
$y=\frac{7}{4}$

j) $105y^3+175y^2-75y=125$
$35y^2(3y+5)-25(3y+5)=0$
$5(7y^2-5)(3y+5)=0$
$y=-\frac{5}{3}$ or $\frac{\sqrt{35}}{7}$

k) $96n^4-84n^3+112n^2=98n$
$12n^3(8n-7)+14n(8n-7)=0$
$n(6n^2+7)(8n-7)=0$
$n=0$ or $\frac{7}{8}$

L) $24x^4+15x^3=56x^2+35x$
$3x^3(8x+5)-7x(8x+5)=0$
$x(3x^2-7)(8x+5)=0$
$x=0$ or $-\frac{5}{8}$ or $\frac{\sqrt{21}}{3}$

3. For what value(s) of "k" can each trinomial be factored?

a. $4x^2+kx+3$
$4\times3=12$
$12=1\times12$
$=2\times6$
$=3\times4$
$=(-3)\times(-4)$
$=(-2)\times(-6)$
$=(-1)\times(-12)$
$\therefore k=13,8,7,-7,-8,-13$

b. $4x^2+kx+25$
$4\times25=100$
$100=1\times100$
$=2\times50$
$=4\times25$
$=5\times20$
$=10\times10$
$k=\pm101,\pm52,\pm29,\pm25,\pm20$

c. $6x^2+kx-9$
$6\times(-9)=-54$
$-54=(-1)\times54=1\times(-54)$
$=(-2)\times27=2\times(-27)$
$=(-3)\times18=3\times(-18)$
$=(-6)\times9=6\times(-9)$
$\therefore k=\pm53,\pm25,\pm16,\pm3$

4. What value of "x" satisfies $x(x-2009)=x(x+2009)$?

$x(x-2009)-x(x+2009)=0.$ $\quad x=0.$

$x(x-2009-x-2009)=0.$

$x\cdot -4018=0.$

5. What are all values of "x" for which $x\sqrt{2}=2\sqrt{x}$?

$\sqrt{2x^2}-\sqrt{2^2x}=0.$ $\quad \therefore x=2 \text{ or } 0.$

$\sqrt{2x}(\sqrt{x}-\sqrt{2})=0.$

$\sqrt{x}=\sqrt{2}.$ or $\sqrt{2x}=0.$

6. What is the only pair of real numbers (a,b) for which the equation is equal $a^3+ab^2=30$ and $b^3+ba^2=90$?

$\begin{cases} a(a^2+b^2)=30 & ① \\ b(a^2+b^2)=90. & ② \end{cases}$

$\frac{②}{①}: \frac{b}{a}=3. \quad b=3a$

$①+② = a(a^2+b^2)+b(a^2+b^2)=120$

$(a+b)(a^2+b^2)=120.$

$(a+3a)(a^2+9a^2)=120$

$4a\cdot 10a^2=120.$

$40a^3=120$

$a^3=3.$

$a=\sqrt[3]{3}.$

$b=3a=3\sqrt[3]{3}$

$\begin{cases} a=\sqrt[3]{3} \\ b=3\sqrt[3]{3} \end{cases}$

7. If r_1, r_2, r_3, r_4 are the roots of $x^4-4x^2+2=0$, what is the value of $(1+r_1)(1+r_2)(1+r_3)(1+r_4)$?

$x^4+0x^3-4x^2+0x+2=0.$

$\begin{cases} r_1+r_2+r_3+r_4=-\frac{0}{1}=0 \\ r_1r_2+r_1r_3+r_1r_4+r_2r_3+r_2r_4+r_3r_4=-4 \\ r_1r_2r_3+r_1r_2r_4+r_2r_3r_4+r_1r_3r_4=0 \\ r_1r_2r_3r_4=2 \end{cases}$

$=(1+r_1+r_2+r_1r_2)(1+r_3+r_4+r_3r_4)$

$=1+r_3+r_4+r_3r_4+r_2+r_2r_3+r_2r_4+r_2r_3r_4$
$+r_1+r_1r_3+r_1r_4+r_1r_3r_4+r_1r_2+r_1r_2r_3+r_1r_2r_4$
$+r_1r_2r_3r_4$

$=1+0+(-4)+0+2.$

$=\boxed{-1}$

8. Factor the following completely: $x^5+x^4+x^3+x^2+x+1$

$=x^3(x^2+x+1)+(x^2+x+1).$

$=(x^3+1)(x^2+x+1).$

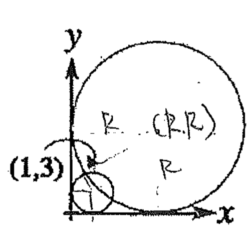
$=(x+1)(x^2-x+1)(x^2+x+1)$

9. Write $4x^2-9y^2+4x^3+6x^2y$ as a product of two non-constant polynomials with integral coefficients.

$=(2x-3y)(2x+3y)+2x^2(2x+3y).$

$=(2x-3y+2x^2)(2x+3y)$

10. Two different circles that pass through the point (1,3) are tangent to both coordinate axes. If the length of the radius of the smaller circle is "r" and the length of the radius of the larger circle is "R", what is the value of "r + R"?



$\sqrt{(R-1)^2+(R-3)^2}=R$

$\sqrt{R^2-2R+1+R^2-6R+9}=R.$

$2R^2-8R+10=R^2$

$R^2-8R+10=0.$

$R=\frac{8\pm\sqrt{64-40}}{2}=4\pm\sqrt{6}$

$\therefore \begin{cases} r=4-\sqrt{6}. \\ R=4+\sqrt{6} \end{cases}$

$r+R=4-\sqrt{6}+4+\sqrt{6}.$

$\therefore \boxed{8}$

11. For what integer "n" are the roots of $x^2 - 7x + n = 0$ consecutive integers?

$-7=(-3)+(-4)$

$(-3)\times(-4)=12$

$\therefore n = \boxed{12}$ //

$x^2-7x+12=0$

$(x-3)(x-4)=0.$

12. If $(x-10)(x+10)=0$, what is the value of $(x-1)(x+1)$?

$x=10$ or -10.

1) $x=10$.

$(x-1)(x+1)$

$=9\cdot 11$

$=99$

2) $x=-10$

$(x-1)(x+1)$

$=-11\cdot -9$

$=99$

$\boxed{99}$ //

13. What is the only value of "x" which satisfies $(x-2005)^2 = (x+2005)^2$?

$x=\boxed{0}$ //

$(-2005)^2=(2005)^2.$

14. Which positive integer "n" satisfies $n^{2006} + 2n^{2007} = 3$

$A=n^{2006}$

$A+2A\cdot n=3$

$1.\ \underline{A}(1+2n)=3.$

$1+2n=3$

$\boxed{n=1}$ //

15. Challenge: Determine all values of "x" for which $(x^2+3x+2)(x^2-2x-1)(x^2-7x+12)+24=0$

$-24=-1\times 2^3\times 3$

$(x^2+3x+2)(x^2-2x-1)(x^2-7x+12)=-24$

$(x+2)(x+1)(x^2-2x-1)(x-3)(x-4)=-24$

$x=2.$ $\quad 4\times 3\times -1\times -1\times -2=-24$

$x=0$ $\quad 2\times 1\times -1\times -3\times -4=-24.$

$x=\boxed{2 \text{ or } 0}$ //

Name: Claire Date: 1.29

Math 9 Enriched: Section 4.5 Factoring Difference and Sums of Powers

Difference of squares: $a^2 - b^2 = (a+b)(a-b)$

Difference and Sums of Cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$ $\quad a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Difference of Powers: $a^n - b^n = (a-b)(a^{n-1} + a^{n-2}b + ... + ab^{n-2} + b^{n-1})$

Difference of Sums: $a^{2n+1} + b^{2n+1} = (a+b)(a^{2n} - a^{2n-1}b + a^{2n-2}b^2 ... - ab^{2n-1} + b^{2n})$

1. Factor each and simplify the following expressions completely:

a) $x^6 - 64$

$=(x^3-8)(x^3+8)$

$=(x-8)(x^2+8x+64)(x+8)(x^2-8x+64)$

b) $9^3 - a^6x^6$

$=2^6 - a^6x^6$

$=(2^3-a^3x^3)(2^3+a^3x^3)$

$=(2-ax)(2+ax)(4+2ax+a^2x^2)(4-2ax+a^2x^2)$

c) $81-(3a+2)^4$

$=[9-(3a+2)^2][9+(3a+2)^2]$

$=(3+3a+2)(3-3a-2)(9+9a^2+12a+4)$

$=(3a+5)(1-3a)(9a^2+12a+13)$

d) $\dfrac{1000+27x^3}{100-9x^2}$

$=\dfrac{\cancel{(10+3x)}(100-30x+9x^2)}{(10-3x)\cancel{(10+3x)}} = \dfrac{100-30x+9x^2}{10-3x}$

e) $\dfrac{a^3-27b^3}{a^2-9b^2}$

$=\dfrac{\cancel{(a-3b)}(a^2+3ab+9b^2)}{\cancel{(a-3b)}(a+3b)}$

$=\dfrac{a^2+3ab+9a^2}{a+3b}$

f) y^6+16y^3+15

$=(y^3+15)(y^3+1)$

$=(y+1)(y^2-y+1)(y^3+15)$

g) x^6-7x^3-8

$x^3 \quad -8$
$x^3 \quad 1$

$=(x^3-8)(x^3+1)$

$=(x-2)(x^2+2x+4)(x+1)(x^2-x+1)$

h) $8y^6-9y^3+1$

$8y^3 \quad -1$
$y^3 \quad -1$

$=(8y^3-1)(y^3-1)$

$=(2y-1)(4y^2+2y+1)(y-1)(y^2+y+1)$

i) x^6-26x^3-27

$=(x^3-27)(x^3+1)$

$=(x-3)(x^2+3x+9)(x+1)(x^2-x+1)$

j) $27y^6+35y^3+8$

$27y^3 \quad 8$
$y^3 \quad 1$

$=(27y^3+8)(y^3+1)$

$=(3y+2)(9y^2-6y+4)(y+1)(y^2-y+1)$

2. Factor the following completely: $x^5+x^4+x^3+x^2+x^1+1=0$

$$x^4(x+1)+x^2(x+1)+(x+1)=0$$
$$(x^4+x^2+1)\ (x+1)=0$$

3. Factor completely: $-a^2b^2+2ab^3-b^4+a^2c^2-2abc^2+b^2c^2$

$$=2ab^3-2abc^2-(a^2b^2-a^2c^2)-(b^4-b^2c^2)$$
$$=2ab(b^2-c^2)-a^2(b^2-c^2)-b^2(b^2-c^2)$$
$$=(2ab-a^2-b^2)(b-c)(b+c)$$
$$=-(a-b)^2(b-c)(b+c)$$

4. Factor completely with integeral coefficients: $x^{12}-y^{12}$

$$=(x^6-y^6)(x^6+y^6)$$
$$=(x^3-y^3)(x^3+y^3)(x^6+y^6)$$
$$=(x-y)(x+y)(x^2+xy+y^2)(x^2-xy+y^2)(x^6+y^6)$$

5. Evaluate : $\dfrac{2^{10}-1}{2^5-1}$

$$=\frac{\cancel{(2^5-1)}(2^5+1)}{\cancel{2^5-1}}$$
$$=2^5+1$$
$$=33$$

6. Factor and simplify the expression as much as possible: $\left(\dfrac{a^3-1}{a^2-1}\right)\left(\dfrac{a^2+2a+1}{a^3+1}\right)\left(\dfrac{a^2-a+1}{a+1}\right)$

$$=\frac{\cancel{(a-1)}(a^2+a+1)}{\cancel{(a-1)}\cancel{(a+1)}}\cdot\frac{\cancel{(a+1)^2}}{(a+1)\cancel{(a^2-a+1)}}\cdot\frac{\cancel{a^2-a+1}}{\cancel{a+1}}$$
$$=\frac{a^2+a+1}{a+1}$$

7. When x^9-x is factored as completely as possible into polynomials and monomials with intral coefficients, how many factors are there?

$$x^9-x$$
$$=x(x^8-1)$$
$$=x(x^4-1)(x^4+1)$$
$$=x(x^2-1)(x^2+1)(x^4+1)$$
$$=x(x-1)(x+1)(x^2+1)(x^4+1)$$

$\boxed{5}$ factors.

8. If $x+y=4$ and $xy=2$, then find x^6+y^6

$$=(x^3+y^3)^2-2x^3y^3$$
$$=(x+y)^2(x^2-xy+y^2)^2-2x^3y^3$$
$$=4^2\cdot[(x+y)^2-3xy]^2-2\cdot2^3$$
$$=16\times64-16$$
$$=\boxed{1008}$$

9. Find the value of $x^6+\frac{1}{x^6}$ if the value of $x+\frac{1}{x}=3$.

$x^2+\frac{1}{x^2}=(x+\frac{1}{x})^2-2$

$=9-2=\boxed{7}$

$x^3+\frac{1}{x^3}=(x+\frac{1}{x})(x^2-1+\frac{1}{x^2})$

$=3\times(7-1)$

$=18.$

$x^6+\frac{1}{x^6}=(x^3+\frac{1}{x^3})^2-2$

$=324-2$

$=\boxed{322}$ //

10. If $a+b=1$, $a^2+b^2=2$, find the value of a^4+b^4

$(a+b)^2=a^2+2ab+b^2$

$1=2+2ab$

$-1=2ab$

$-\frac{1}{2}=ab$

$a^4+b^4=(a^2+b^2)^2-2a^2b^2$

$=4-2\cdot\frac{1}{4}$

$=\boxed{\frac{7}{2}}$ //

11. If $\frac{1}{a+c}=\frac{1}{a}+\frac{1}{c}$, find the value of $\left(\frac{a}{c}\right)^3$.

$\frac{1}{a+c}=\frac{a+c}{ac}$

$a^2+2ac+c^2=ac$

$a^2+ac+c^2=0$

$(a-c)(a^2+ac+c^2)=0$

$a^3-c^3=0$

$a^3=c^3$.

$\left(\frac{a}{c}\right)^3=\frac{a^3}{c^3}=\frac{a^3}{a^3}=\boxed{1}$ //

12. Find the sum of $\frac{1}{3+2\sqrt{2}}+\frac{1}{2\sqrt{2}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+2}+\frac{1}{2+\sqrt{3}}$.

$=\frac{1}{\sqrt{9}+\sqrt{8}}+\frac{1}{\sqrt{8}+\sqrt{7}}+\frac{1}{\sqrt{7}+\sqrt{6}}+\frac{1}{\sqrt{6}+\sqrt{5}}+\frac{1}{\sqrt{5}+\sqrt{4}}+\frac{1}{\sqrt{4}+\sqrt{3}}$

$=\frac{9-8}{\sqrt{9}+\sqrt{8}}+\frac{8-7}{\sqrt{8}+\sqrt{7}}+\cdots+\frac{4-3}{\sqrt{4}+\sqrt{3}}$

$=\frac{(\sqrt{9}-\sqrt{8})\cancel{(\sqrt{9}+\sqrt{8})}}{\cancel{\sqrt{9}+\sqrt{8}}}+\cdots+\frac{(\sqrt{4}-\sqrt{3})\cancel{(\sqrt{4}+\sqrt{3})}}{\cancel{\sqrt{4}+\sqrt{3}}}=\sqrt{9}-\cancel{\sqrt{8}}+\cancel{\sqrt{8}}-\sqrt{7}+\cdots\sqrt{4}-\sqrt{3}$.

$=\boxed{3-\sqrt{3}}$ //

13. Challenge: Find the sum of: $\frac{1}{\sqrt[3]{1}+\sqrt[3]{2}+\sqrt[3]{4}}+\frac{1}{\sqrt[3]{4}+\sqrt[3]{6}+\sqrt[3]{9}}+\frac{1}{\sqrt[3]{9}+\sqrt[3]{12}+\sqrt[3]{16}}$

$=\frac{\sqrt[3]{1}-\sqrt[3]{2}}{(\sqrt[3]{1})^3-(\sqrt[3]{2})^3}+\frac{\sqrt[3]{2}-\sqrt[3]{3}}{(\sqrt[3]{2})^3-(\sqrt[3]{3})^3}+\frac{\sqrt[3]{3}-\sqrt[3]{4}}{(\sqrt[3]{3})^3-(\sqrt[3]{4})^3}$

$=\sqrt[3]{2}-\sqrt[3]{1}-\sqrt[3]{2}+\sqrt[3]{3}-\sqrt[3]{3}+\sqrt[3]{4}$.

$=\boxed{2^{\frac{2}{3}}-1}$ //.

14. Show that $2^{99}+3^{99}$ is divisible by 35.

$=(2^3)^{33}+(3^3)^{33}$

$=\underbrace{(2^3+3^3)}_{}[(2^3)^{32}-(2^3)^{31}(3^3)+\cdots+(3^3)^{32}]$

$8+27=\boxed{35}$ //

Name: Claire Li　　　　Date: 1.31

Math 9 Enriched HW 4.6 Solving Equations by CTS

1. Indicate what value should be added to the trinomial so that the equation could be a perfect trinomial:

a) $x^2+(?)+9$ 3 $2\cdot x\cdot 3=\boxed{6x}$	b) $x^2+8x+(?)$ $4^2=\boxed{16}$ $\frac{8x}{2x}=4$
c) $(?)-2x+1$ $\boxed{x^2}$	d) $x^2-(?)+81$ $2\cdot 9\cdot x=\boxed{18x}$
e) $x^2-15x+(?)$ $\left(\frac{15}{2}\right)^2\boxed{\frac{225}{4}}$	f) $x^2+17x+(?)$ $\left(\frac{17}{2}\right)^2=\boxed{\frac{289}{4}}$
g) $4x^2+4x+(?)$ $\frac{4x}{2\cdot 2x}$ $\boxed{1}$	h) $9x^2-(?)+1$ $2\cdot 3x\cdot 1=\boxed{6x}$

2. Solve each of the following equations algebraically:

a) $(x-3)^2-12=0$ $(x-3)^2=12$ $x=3\pm2\sqrt{3}$	b) $(2x+4)^2-16=0$ $2^2(x+2)^2=16$ $(x+2)^2=4$ $=2\pm4$	c) $-4(x+7)^2+14=0$ $2(x+7)^2=7$ $x+7=\pm\sqrt{\frac{7}{2}}$ $x=-7\pm\frac{\sqrt{14}}{2}$
d) $0.5(x+11)^2-11=0$ $\frac{1}{2}(x+11)^2=11$ $x+11=\pm\sqrt{22}$ $x=-11\pm\sqrt{22}$	e) $(x+5)^2+12=0$ $(x+5)^2=-12$ No solution.	f) $\frac{(2x+1)^2}{3}-15=0$ $(2x+1)^2=45$ $2x+1=\pm3\sqrt{5}$ $2x=-1\pm3\sqrt{5}$ $x=-\frac{1}{2}\pm\frac{3\sqrt{5}}{2}$

3. Solve each of the following quadratic equations by Completing the Square. Please show all your steps:

a) $0=2x^2+11x-5$ $0=x^2+\frac{11}{2}x-\frac{5}{2}$ $0=\left(x+\frac{11}{4}\right)^2-\frac{161}{16}$ $\frac{161}{16}=\left(x+\frac{11}{4}\right)^2$ $\pm\frac{\sqrt{161}}{4}=x+\frac{11}{4}$ $-\frac{11}{4}\pm\frac{\sqrt{161}}{4}=x$	b) $0=0.5x^2+6x-3$ $0=x^2+12x-6$ $0=(x+6)^2-42$ $42=(x+6)^2$ $\pm\sqrt{42}=x+6$ $-6\pm\sqrt{42}=x$

c) $3x^2 = 2 - 15x$ $3x^2+15x-2=0$ $3(x^2+5x)-2=0$ $3(x+\frac{5}{2})^2-\frac{83}{4}=0$ $(x+\frac{5}{2})^2=\frac{83}{12}$ $x+\frac{5}{2}=\pm\frac{\sqrt{83}}{2\sqrt{3}}$ $x=-\frac{5}{2}\pm\frac{\sqrt{249}}{6}$	d) $0 = -\frac{2}{3}x^2 + 12x + 3$ $0=-2x^2+36x+9$ $0=x^2-18x-\frac{9}{2}$ $0=(x-9)^2-81-\frac{9}{2}$ $\frac{171}{2}=(x-9)^2$ $\pm\frac{3\sqrt{38}}{2}=x-9$ $x=9\pm\frac{3\sqrt{38}}{2}$
d) $0 = -\frac{3}{2}x^2 - 8x + 9$ $0=3x^2+16x-18$ $0=x^2+\frac{16}{3}x-6$ $0=(x+\frac{8}{3})^2-\frac{64}{9}-\frac{54}{9}$ $(x+\frac{8}{3})^2=\frac{118}{9}$ $x+\frac{8}{3}=\pm\frac{\sqrt{118}}{3}$ $x=-\frac{8}{3}\pm\frac{\sqrt{118}}{3}$	d) $0 = -\frac{4}{5}x^2 + 10x + 11$ $0=4x^2-50x-55$ $0=(2x-\frac{25}{2})^2-\frac{625}{4}-55$ $0=(2x-\frac{25}{2})^2-\frac{845}{4}$ $\pm\frac{13\sqrt{5}}{2}=2x-\frac{25}{2}$ $2x=\frac{25}{2}\pm\frac{13\sqrt{5}}{2}$ $x=\frac{25\pm13\sqrt{5}}{4}$

4. The sum of an arithmetic series is given by the equation: $S = \frac{n}{2}(2 \times a + [n-1]d)$, where "n" is the number of terms, "a" is the first term, and "d" is the common difference. If the first term *"a"* is 10, common difference *"d"* is 4, and the sum *"S"* is 1144, find the number of terms *"n"* in the series.

$1144=\frac{n}{2}(2\times10+(n-1)4)$

$1144=\frac{n}{2}(20+4n-4)$

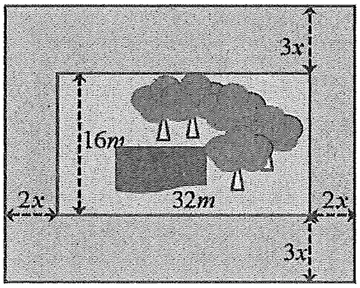
$1144=8n+2n^2$

$0=n^2+4n-572$

$0=(n+26)(n-22)$

$n=22\ (n\neq-26)$

5. A rectangular playground (16m by 32m) has a walkway around it as shown below. If adding the walkway doubles the area of the playground, find the value of "x":



$16+6x$

$4x+32$

$(16+6x)(4x+32) = 2\cdot 16\cdot 32$

$(8+3x)(x+8) = 128$

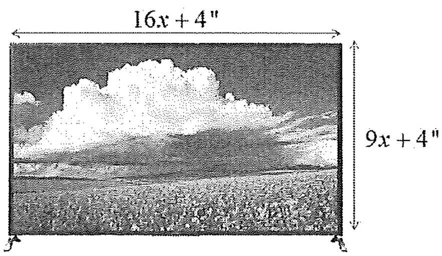
$3x^2+24x+8x+64 = 128$

$3x^2+32x-64 = 0$

$x = \frac{-32 \pm \sqrt{1024+768}}{6} = \frac{-16 \pm 8\sqrt{7}}{3}$

$\boxed{x = \frac{-16+8\sqrt{7}}{3}}$

6. Jason bought a 75" television at Costco. He knows that the screen aspect ratio is 16:9 [width to height]. Besides the screen, there is also a uniform border of 2" around. What is the width of the TV?



$(16x+4)(9x+4) = 75$

$144x^2+100x+16 = 75$

$144x^2+100x-59 = 0$

$(12x+\frac{25}{6})^2 - \frac{625}{36} - 59 = 0$

$(12x+\frac{25}{6})^2 = \frac{2749}{36}$

$12x+\frac{25}{6} = \pm\frac{\sqrt{2749}}{6}$

$x = -\frac{25}{72} + \frac{\sqrt{2749}}{72}$

7. A number is 4 more than its reciprocal. Find the number

$\frac{1}{x}+4 = x$

$1+4x = x^2$

$0 = x^2-4x-1$

$0 = (x-2)^2-5$

$\pm\sqrt{5} = x-2$

$\boxed{x = 2\pm\sqrt{5}}$

8. The sum of a number and its reciprocal is 3. What is the number?

$(x+\frac{1}{x}) = 3$

$x^2+1 = 3x$

$x^2-3x+1 = 0$

$(x-\frac{3}{2})^2 - \frac{9}{4}+1 = 0$

$(x-\frac{3}{2})^2 = \frac{5}{4}$

$x-\frac{3}{2} = \pm\frac{\sqrt{5}}{2}$

$x = \frac{3}{2} \pm \frac{\sqrt{5}}{2}$

$\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

9. Find a positive number whose square is 3 more than the number itself

$x^2-3 = x$

$x^2-x-3 = 0$

$x = \frac{1\pm\sqrt{1+12}}{2} = \frac{1\pm\sqrt{13}}{2}$

$\boxed{x = \frac{1+\sqrt{13}}{2}}$

10. Find a negative number whose square is 29 more than the number itself

$x^2 - 29 = x$

$x^2 - x - 29 = 0$

$(x-\frac{1}{2})^2 - \frac{1}{4} - 29 = 0$

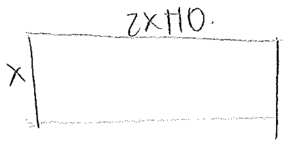
$(x-\frac{1}{2})^2 = 29\frac{1}{4}$

$x - \frac{1}{2} = \pm\frac{3\sqrt{13}}{2}$

$x = \frac{1 \pm 3\sqrt{13}}{2}$

negative: $\frac{1-3\sqrt{13}}{2}$

11. The length of a rectangle is 10cm more than the twice the width. The area is 120cm2. What are the dimensions of the rectangle to the nearest tenth?



$(2x+10)x = 120$

$2x^2 + 10x = 120$

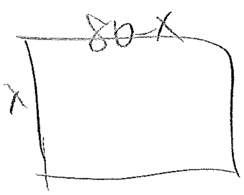
$x^2 + 5x - 60 = 0$

$x = \frac{-5 \pm \sqrt{25+240}}{2} = \frac{-5+\sqrt{265}}{2} \quad \left(\frac{-5-\sqrt{265}}{2}\right)$

$-5+\sqrt{265}+10 = \sqrt{265}+5$

$\begin{cases} \text{width} = \frac{-5-\sqrt{265}}{2} \\ \text{length} = \sqrt{265}+5 \end{cases}$

12. A picture frame has an area of 340cm2 and a perimeter of 80cm. What are the dimensions of the picture frame?



$x(80-x) = 340$

$80x - x^2 = 340$

$x^2 - 80x + 340 = 0$

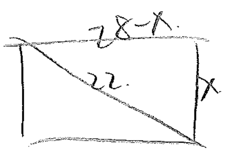
$(x-40)^2 - 1600 + 340 = 0$

$x - 40 = \pm 6\sqrt{35}$

$x = 6\sqrt{35} + 40$

$\begin{cases} 6\sqrt{35}+40 \\ 40 - 6\sqrt{35} \end{cases}$

13. Find the sides of a rectangle whose perimeter is 56 and diagonal is 22



$(28-x)^2 + x^2 = 22^2$

$x^2 - 56x + 776 + x^2 = 484$

$2x^2 - 56x - 292 = 0$

$x^2 - 28x - 146 = 0$

$(x-14)^2 - 196 - 146 = 0$

$(x-14)^2 = 342$

$x - 14 = \pm 3\sqrt{38}$

$x = 14 + 3\sqrt{38} \quad (14 - 3\sqrt{38})$

14. Complete the square and isolate the variable "x": $mx^2 + nx + p = 0$

$x^2 + \frac{n}{m}x + \frac{p}{m} = 0$

$(x+\frac{n}{2m})^2 - \frac{n^2}{4m^2} + \frac{p}{m} = 0$

$(x+\frac{n}{2m})^2 = \frac{n^2}{4m^2} - \frac{4mp}{4m^2}$

$x + \frac{n}{2m} = \pm\frac{\sqrt{n^2-4mp}}{2m}$

$x = \frac{-n \pm \sqrt{n^2-4mp}}{2m}$

Name: Claire Li

Date: 2.4

Math 10 Honors Section 4.7 Quadratic Formula:

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a} \quad , \quad a\neq 0$$

1. Solve for "x" for each of the following:

a) $2x^2-3x-1=0$ $x=\frac{3\pm\sqrt{9+8}}{4}=\boxed{\frac{3\pm\sqrt{17}}{4}}$	b) $3x^2-x+1=3$ $3x^2-x-2=0$ $x=\frac{1\pm\sqrt{1+24}}{6}=\frac{1\pm5}{6}=\boxed{1 \text{ or } -\frac{2}{3}}$	c) $x^2-5x+6=0$ $x=\frac{5\pm\sqrt{25-24}}{2}=\frac{5\pm1}{2}=\boxed{2 \text{ or } 3}$
d) $2x^2+3x+8=0$ $x=\frac{-3\pm\sqrt{9-64}}{4}$ No real solution	e) $x-1=\frac{1}{2x}$ $2x^2-2x=1$ $2x^2-2x-1=0$ $x=\frac{2\pm\sqrt{4+8}}{4}$ $=\frac{1\pm2\sqrt{3}}{2}$	f) $3x-1=\frac{1}{x}$ $3x^2-x=1$ $3x^2-x-1=0$ $x=\frac{1\pm\sqrt{1+12}}{6}$ $=\frac{1\pm\sqrt{13}}{6}$
g) $2x^3-5x^2+7x=0$ $x(2x^2-5x+7)=0$ $\boxed{x=0}$ $x=\frac{5\pm\sqrt{25-56}}{4}$	h) $\frac{4x^2}{3}=4x-2$ $4x^2-12x+6=0$ $2x^2-6x+3=0$ $x=\frac{6\pm\sqrt{36-24}}{4}$ $=\frac{3\pm\sqrt{3}}{2}$	i) $x^2-2=\frac{-7x}{2}$ $2x^2+7x-4=0$ $x=\frac{-7\pm\sqrt{49+32}}{4}$ $=\frac{-7\pm9}{4}=\frac{1}{2}$ or -4
j) $5x-1=2x^2$ $0=2x^2-5x+1$ $x=\frac{5\pm\sqrt{25-8}}{4}$ $=\frac{5\pm\sqrt{17}}{4}$	k) $8x-9=10x^2$ $0=10x^2-8x+9$ $x=8\pm\sqrt{64-360}$ No real solution.	L) $9x^2+6x+1=0$ $(3x+1)^2=0$ $3x+1=0$ $x=-\frac{1}{3}$
m) $3x(2x-6)=8$ $6x^2-18x-8=0$ $3x^2-9x-4=0$ $x=\frac{9\pm\sqrt{81+48}}{6}$ $=\frac{9\pm\sqrt{129}}{6}$	n) $2(2x-1)^2+9(2x-1)+7=0$ $(2(2x+1)+7)(2x+1+1)=0$ $(4x-5)(2x+8)=0$ $x=\frac{5}{4}$ or -4	O) $2x^2+6x-8=7x^2-2x$ $-5x^2+8x-8=0$ $5x^2-8x+8=0$ $\sqrt{64-160}$ No real solution.
p) $2x(x-4)=-7$ $2x^2-8x=-7$ $2x^2-8x+7=0$ $x=\frac{8\pm\sqrt{64-56}}{4}=\frac{4\pm\sqrt{2}}{2}$	q) $4x(3x-4)=1$ $12x^2-16x-1=0$ $x=\frac{16\pm\sqrt{256+48}}{24}$ $=\frac{4\pm\sqrt{19}}{6}$	r) $0.3x^2-0.5x-1=0.25$ $30x^2-50x-100=.25$ $6x^2-10x-21=0$ $x=10\pm\sqrt{100-}$
s) $(x+5)(x-4)=2(x+1)$ $x^2+x-20=2x+2$ $x^2-x-22=0$ $x=\frac{1\pm\sqrt{1+88}}{2}=\frac{1\pm\sqrt{89}}{2}$	t) $\frac{3}{x}-2=\frac{4}{x+2}$ $3(x+2)-2x^2-4x=4x$ $3x+6-2x^2-4x=4x$ $-2x^2-8x+3x=0$ $2x^2+5x-12=0$ $x=\frac{-5\pm\sqrt{25+96}}{4}$ $=\frac{-5\pm11}{4}$ $x=\frac{3}{2}$ or -4.	u) $\frac{3}{x+2}+\frac{2}{x-3}=2$ $3(x-3)+2(x+2)=2(x+2)(x-3)$ $3x-9+2x+4=2(x^2-x-6)$ $5x-5=2x^2-2x-12$ $2x^2-7x-7=0$ $x=\frac{7\pm\sqrt{49+56}}{4}$ $=\frac{7\pm\sqrt{105}}{4}$

2. Under what conditions will the equation have no solutions? $ax^2+bx+c=0$ Explain why

when $b^2-4ac<0$

because $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ if $b^2-4ac<0$

x has no real solutions.

3. Under what conditions will the equation have only one solution? $ax^2+bx+c=0$ Explain why

when $b^2-4ac=0$.

$x=\frac{-b}{2a}$.

4. If the distance between the points A(2,3) and B(a, 3a) is 10 units long. What are all the possible values of "a"?

$\sqrt{(2-a)^2+(3-3a)^2}=10$

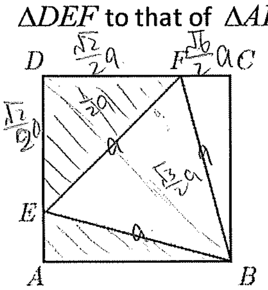
$a^2-4a+4+9a^2-18a+9=100$

$10a^2-22a-87=0$

$a=\frac{22\pm\sqrt{484+3480}}{20}$

$=\frac{22\pm2\sqrt{991}}{20}=\frac{11\pm\sqrt{991}}{10}$

5. Points E and F are located on square ABCD so that ΔBEF is equilateral. What is the ratio of the area of ΔDEF to that of ΔABE



$\frac{\left(\frac{1}{2}a+\frac{\sqrt{3}}{2}a\right)}{\sqrt{2}}=\frac{1+\sqrt{3}}{2}a\cdot\frac{1}{\sqrt{2}}=\frac{\sqrt{2}+\sqrt{6}}{2}a.$

Area$\Delta DEF=\frac{\sqrt{2}}{2}a\cdot\frac{\sqrt{2}}{2}a\cdot\frac{1}{2}=\frac{1}{4}a^2.$

Area$\Delta ABE=\frac{\sqrt{6}}{2}a\cdot\frac{(\sqrt{2}+\sqrt{6})a}{2}\cdot\frac{1}{2}=\frac{\sqrt{3}+2}{4}a^2$

$\frac{\frac{1}{4}a^2}{\frac{\sqrt{3}+2}{4}a^2}=\frac{1}{\sqrt{3}+2}=\boxed{2-\sqrt{3}}$

6. The quadratic equation $x^2+mx+n=0$ has roots that are twice those of $x^2+px+m=0$, and none of m,n, and p is zero. What is the value of n/p?

$m=2p.$

$\frac{-m\pm\sqrt{m^2-4n}}{2}=2\cdot\frac{-p\pm\sqrt{p^2-4m}}{2}$

$-m\pm\sqrt{m^2-4n}=-2p\pm2\sqrt{p^2-4m}$

$\sqrt{4p^2-4n}=2\sqrt{p^2-8p}$

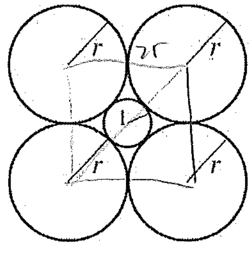
$\sqrt{4p^2-4n}=\sqrt{4p^2-32p}.$

$4p^2-4n=4p^2-32p.$

$n=8p.$

$\frac{n}{p}=\boxed{8}$

7. A circle of radius 1 is surrounded by 4 circles of radius "r" as shown. What is r?



$(2+2r)=2r\cdot\sqrt{2}.$

$(r+1)=\sqrt{2}r.$

$r^2+2r+1=2r^2$

$0=r^2-2r-1$

$r=\frac{2\pm\sqrt{4+4}}{2}=\frac{2\pm2\sqrt{2}}{2}=1\pm\sqrt{2}.$

$\boxed{1+\sqrt{2}}$

8. There are two values of "a" for which the equation $4x^2+ax+8x+9=0$ has only one solution for "x". What is the sum of those values of "a"?

$4x^2+(a+8)+9=0$

(A) -16 (B) -8 (C) 0 (D) 8 (E) 20

$\to b^2-4ac=0$

$(a+8)^2-4\times9\times4=0$

$(a+8)^2=12^2$

$a+8=\pm12$

$a=4$ or -20

$4+(-20)=-16$

9. Both roots of the quadratic equation are prime numbers. $x^2-63x+k=0$. What are all the possible values of k"?

$k=122$

$k=\boxed{122}$

10. Two different positive numbers "a" and "b" each differ from their reciprocals by 1. What is "a+b"?

$a-\frac{1}{a}=1$

$a^2-a-1=0$

$a=\frac{1+\sqrt{5}}{2}$

$b=\frac{1}{a}=1+\frac{2}{1\pm\sqrt{5}}=\frac{2}{1+\sqrt{5}}=\frac{2-2\sqrt{5}}{1-5}=\frac{\sqrt{5}-1}{2}$

$a+b=\frac{1+\sqrt{5}}{2}+\frac{\sqrt{5}-1}{2}=\frac{2\sqrt{5}}{2}=\boxed{\sqrt{5}}$

11. There are two values of "a" for which the equation $4x^2+ax+8x+9=0$ has only one solution for "x". What are these two values of "a"?

$\Delta=0$

$(8+a)^2-4\times36=0$

$8+a=\pm12$

$a=\boxed{4 \text{ or } -20}$

12. (Challenge) How many pairs of positive integers (a,b) are there such that "a" and "b" have no common factors greater than 1 and $\frac{a}{b}+\frac{14b}{9a}$ is an integer?

a) 4 b) 6 c) 9 d) 12 e) Infinitely many f) none

$\frac{9a^2+14b^2}{9ab}$

Name: Claire Li Date:________________

Math 10 Honors Section 4.8 Long Division:

1. Divide and find the quotient. Write the division statement for each of the following:

i) $\dfrac{x^2-3x+5}{x-2}$

$x-1$

$x-2\,)\,x^2-3x+5$

x^2-2x

$-x+5$

$-x+2$

3

$x^2-3x+5=(x-2)(x-1)+3$

ii) $\dfrac{x^3-5x^2+10x-15}{x-3}$

x^2-2x+4

$x-3\,)\,x^3-5x^2+10x-15$

x^3-3x^2

$-2x^2+10x$

$-2x^2+6x$

$4x-15$

$4x-12$

-3

$x^3-5x^2+10x-15=(x-3)(x^2-2x+4)-3$

iii) $\dfrac{x^3+13x^2+39x+20}{x+9}$

x^2+4x+3

$x+9\,)\,x^3+13x^2+39x+20$

x^3+9x

$4x^2+39x$

$4x^2+36x$

$3x+20$

$3x+27$

-7

$x^3+13x^2+39x+20=(x+9)(x^2+4x+3)-7$

iv) $\dfrac{x^3-12x-20}{x+2}$

x^2-2x-8

$x+2\,)\,x^3+0x^2-12x-20$

x^3+2x^2

$-2x^2-12x$

$-2x^2-4x$

$-8x-20$

$-8x-16$

-4

$x^3-12x-20=(x+2)(x^2-2x-8)-4$

v) $\dfrac{x^4+x^2-3x-1}{2x+1}$

$\frac{1}{2}x^3-\frac{1}{4}x^2+\frac{5}{8}x-\frac{29}{16}$

$2x+1\,)\,x^4+0x^3+x^2-3x-1$

$x^4+\frac{1}{2}x^3$

$-\frac{1}{2}x^3+x^2$

$-\frac{1}{2}x^3-\frac{1}{4}x^2$

$\frac{5}{4}x^2-3x$

$\frac{5}{4}x^2+\frac{5}{8}x$

$-\frac{29}{8}x-1$

$-\frac{29}{8}x-\frac{29}{16}$

$\frac{13}{16}$

$x^4+x^2-3x-1=(2x+1)(\frac{1}{2}x^3-\frac{1}{4}x^2+\frac{5}{8}x-\frac{29}{16})+\frac{13}{16}$

vi) $\dfrac{2x^3-13x+5x^2-28}{x+3}$

$2x^2-x-10$

$x+3\,)\,2x^3+5x^2-13x-28$

$2x^3+6x^2$

$-x^2-13x$

$-x^2-3x$

$-10x-28$

$-10x-30$

2

$2x^3-13x+5x^2-28=(x+3)(2x^2-x-10)+2$

vii) $\dfrac{32x^3-18x+16x^2+9}{2x^2+1}$

$16x+8$

$2x^2+0x+1\,)\,32x^3+16x^2-18x+9$

$32x^3+0+16x$

$16x^2-34x+9$

$16x^2+0+8$

$-34x+1$

$32x^3-18x+16x^2+9=(2x^2+1)(16x+8)-34x+1$

viii) $\dfrac{-2x^3+4x^2-3x+5}{4x^2+5}$

$-\frac{1}{2}x+1$

$4x^2+5\,)\,-2x^3+4x^2-3x+5$

$-2x^3+0-\frac{5}{2}x$

$4x^2-\frac{1}{2}x+5$

$4x^2+0+5$

$-\frac{1}{2}x$

$-2x^3+4x^2-3x+5=(4x^2+5)(-\frac{1}{2}x+1)-\frac{1}{2}x$

ix) $\dfrac{2x^3+8x^4-3x^2+1+7x}{4x+1}$

$2x^3+0-\frac{3}{4}x+\frac{31}{16}$

$4x+1\,)\,8x^4+2x^3-3x^2+7x+1$

$8x^4+2x^3$

$-3x^2+7x$

$-3x^2-\frac{3}{4}x$

$\frac{31}{4}x+1$

$\frac{31}{4}x+\frac{31}{16}$

$-\frac{15}{16}$

$2x^3+8x^4-3x^2+1+7x=(4x+1)(2x^3-\frac{3}{4}x+\frac{31}{16})-\frac{15}{16}$

$\dfrac{x^4-1}{x-1}=\dfrac{(x^2-1)(x^2+1)}{x-1}$

$=\dfrac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{x-1}}$

$=(x+1)(x^2+1)$

$=x^3+x^2+x+1$

$x^4-1=(x-1)(x^3+x^2+x+1)$

$\dfrac{x^4+x^3+x^2+x+1}{x+1}$

x^3+0+x

$x+1\,)\,x^4+x^3+x^2+x+1$

x^4+x^3

x^2+x

x^2+x

1

$x^4+x^3+x^2+x+1=(x+1)(x^3+x)+1$

$\dfrac{9x^2-5}{3x+2}$

$3x-2$

$3x+2\,)\,9x^2+0x-5$

$9x^2+6x$

$-6x-5$

$-6x-4$

-1

$9x^2-5=(3x+2)(3x-2)-1$

2. Determine the value of "k" such that when $2x^3+9x^2+kx-15$ is divided by $x+5$, the remainder is 0.

$2x^2-x-3$

$x+5\,)\,2x^3+9x^2+kx-15$

$2x^3+10x^2$

$-x^2+kx$

$-x^2-5x$

-15

-15

$kx-(-5x)=-3x$

$kx+5x=-3x$

$kx=-8x$

$k=\boxed{-8}$

3. Divide: $3x^4-6x^3+9x^2-5x+8$ by x^2+x-3

$3x^2-3x+15$

$x^2-x-3\,)\,3x^4-6x^3+9x^2-5x+8$

$3x^4-3x^3-9x^2$

$-3x^3+18x^2-5x$

$-3x^3+3x^2+9x$

$15x^2-14x+8$

$15x^2-15x-45$

$x+53$

$3x^4-6x^3+9x^2-5x+8=(x^2+x-3)(x^2-x-3)+(x+53$

4. Divide: $10x^4+8x^3-9x^2+7x-3$ by x^2+x-3

$10x^4+8x^3-9x^2+7x-3$

$=(x^2+x-3)(10x^2-2x+23)-(22x+66)$

$10x^2-2x+23$

$x^2+x-3\,)\,10x^4+8x^3-9x^2+7x-3$

$10x^4+10x^3-30x^2$

$-2x^3+21x^2+7x$

$-2x^3-2x^2+6x$

$23x^2+x-3$

$23x^2+23x-69$

$-22x+66$

5. Divide: $8x^3-6x^2+2x-5$ by $4x^2-x-1$

$2x-1$

$4x^2-x-1\,)\,8x^3-6x^2+2x-5$

$8x^3-2x^2-2x$

$-4x^2+4x-5$

$-4x^2+x+1$

$3x-6$

6. When $2x^2+x-7$ is divided by $ax+b$, the quotient is $2x+5$ and the remainder is 3. What are the values of "a" and "b"?

$2x^2+x-7=(2x+5)(ax+b)+3$

$2x^2+x-10=(2x+5)(ax+b)$

$(x-2)(2x+5)+3$

$\begin{cases} a=1 \\ b=-2 \end{cases}$

7. If ax^2+bx+1 is divded by $2x-3$, the remainder is 0. What is the value of $3a+2b$?

$3/2\;|\;a \quad b \quad 1$

$-b-\frac{2}{3} \quad -1$

$-\frac{2}{3}b-\frac{4}{9} \quad -2/3 \quad 0$

$a=-\frac{2}{3}b-\frac{4}{9}$

$3a+2b$

$=-2b-\frac{4}{3}+2b$

$=-\frac{4}{3}$

8. Given the expression: $2x^3-3x^2-8x-3$, which of the following will give a remainder of 0 when divided by it: $x+1$, $x-1$, or $x-3$?

-1 or 1 or 3 (a)

$|\;2 \quad -3 \quad -8 \quad -3$

$2a \quad 2a^2-3a \quad 3$

$2 \quad 2a-3 \quad 0$

$(2a^2-3a-8)a=3$

$a=1 - 2-3-8$ ✗

$a=-1 - -2+3-8$ ✗

$a=3 - 18-9-8$ ✓

$\boxed{x-3}$

9. When x^2-3x+k is divided by $x-k$, the remainder is "k". Find all the possible the value(s) of "k".

$$\begin{array}{c|ccc} k & 1 & -3 & k \\ & & k & 0 \\ \hline & 1 & k-3 & k \end{array}$$

$k^2-3k=0$

$k(k-3)=0$

$\boxed{k = 0 \text{ or } 3}$

10. Divide and find the quotient for each of the following. Then factor the quotient.

11. Given that $x^3-x^2-24x-36=(x+3)(x+A)(x+B)$. What are the values of "A" and "B"?

$$\begin{array}{c|cccc} -3 & 1 & -1 & -24 & -36 \\ & & -3 & 12 & 36 \\ \hline & 1 & -4 & -12 & 0 \end{array}$$

$=(x+3)(x^2-4x-12)$

$=(x+3)(x-6)(x+2)$

$\begin{cases} A=-6/2 \\ B=2/-6 \end{cases}$

12. Given that $6x^3-11x^2-4x+4=(x-3)(2x+A)(3x+B)$. What are the values of "A" and "B"?

$6x^2+7x+17$

$x-3\,)\,6x^3-11x^2-4x+4$

$6x^3-18x^2$

$7x^2-4x$

$7x^2-21x$

$17x+4$

$17x-51$

55

$=(x-3)(6x^2+7x+17)$

no solution.

13. When the dividend ax^3+bx^2+4x-8 is divided by x^2-x-2, the remainder is $15x+2$. Find the values of "a" and "b".

$ax+(b+a)$

$x^2-x-2\,)\,ax^3+bx^2+4x-8$

ax^3-ax^2-2ax

$(b+a)x^2+(4+2a)x-8$

$(b+a)x^2-(a+b)x-2a-2b$

$\begin{cases} 4+2a+a+b=15 \\ -8-2a+2b=2 \end{cases} \rightarrow \begin{cases} 3a+b=11 \\ 2a+2b=10 \end{cases}$

$\therefore \begin{cases} a=3 \\ b=2 \end{cases}$

14. When the dividend $ax^3+bx^2-53x-8$ is divided by x^2-5x-6, the remainder is $48x-86$. Find the values of "a" and "b".

$15x+2$

$ax+(b+5a)$

$x^2-5x-6\,)\,ax^3+bx^2-53x-8$

ax^3-5ax^2-6ax

$(b+5a)x^2+(6a-53)x-8$

$(b+5a)x^2+(-5b-25a)x-6b-30a$

$6a-53 \quad -8+6b$

$+5b+25a \quad +30a$

$48x-84$

$\begin{cases} 31a+5b=101 \\ 30a+6b=-78 \end{cases}$

$\therefore \begin{cases} a=\frac{83}{3} \\ b=\frac{-454}{3} \end{cases}$